SOLAR CORONA
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Comments on Coronal Heating

E. N. PARKER *

Enrico Fermi Institute for Nuclear Studies and Department of Physics, University of Chicago, Chicago, Illinois

It is generally agreed that mechanical motions generated in the heat engine represented by the convective zone beneath the photosphere are responsible for the heating of the solar chromosphere and corona. There are several modes generated in the convective zone which are capable of propagation into the solar corona, and there are several mechanisms operative in the corona to dissipate these motions and generate the necessary heat. The first propagating mode to be discussed in this connection seems to have been the hydromagnetic wave (*1*). The objection at first was that no dissipation mechanism was known. More recently it has been shown that hydromagnetic waves will steepen into shocks and dissipate (see, for instance, Osterbrock, *2*) satisfactorily. The next mode to be considered was the acoustical mode (*3, 4*), which dissipated in the chromosphere and corona because of the tendency to steepen into shocks (*5, 6*). The third mode to be considered was the internal gravity wave, suggested by Hines (*7*). Recent work by Whitaker (*8*) shows that the convective stability of the photospheric layers into which the granules extend is such that the upper portions of the granules must be largely gravity waves, implying that the generation of gravity waves by the granules has an efficiency of the order of 0.5. Using the recent granule studies of Bahng and Schwarzschild (*9*), Whitaker has shown the gravity waves account for the million degree solar corona. The dissipation of the gravity waves is principally through thermal conductivity, so they dissipate mainly in the corona. The calculations also show that only the high frequency end of the acoustical wave spectrum from the granules can penetrate through the more stable layers above the photosphere, suggesting that acoustical waves may be relatively unimportant and the chromosphere may be heated largely by thermal conduction downward from the base of the corona.

Finally we should like to point out a mechanism for coronal heating which, though certainly not the principal source, is nonetheless not a trivial one and seems so far to have been generally overlooked: The helium ions in a well stirred corona may contribute a significant amount of heating as a consequence of settling downward through the hydrogen. The remainder of this paper is devoted to this topic.

The discussion begins with a consideration of the hydrostatic equilibrium of an atmosphere composed of more than one kind of ion. This question was considered many years ago by Pannekoek (*10*), Milne (*11*), Rosseland (*12*), and Fowler (*13*) in connection with the structure of stellar atmospheres. We would like to reopen the discussion with particular attention to conditions in the solar corona, emphasizing the rapid tendency toward stratification of ionic species with different charge to mass ratio in the absence of sufficiently vigorous stirring.* The discussion of stratified hydrostatic equilibrium serves as a starting point for discussion of coronal heating by helium drift.

Consider an atmosphere composed of n distinct ion species, with individual charge Z_ie and mass A_iM $(i = 1,2,\cdots n)$. The equilibrium distribution of these ions is independent of any neutral atoms which may be present. We restrict the calculations to the simple case of an isothermal atmosphere. Generalization to more general circumstances is straightforward, but computationally rather more complex. We represent distance measured upward in the gravitational field by s, denoting the downward gravitational acceleration by $g(s)$. Denoting the number of ions per unit volume for the i'th species by N_i, it follows that the electron density N_e is

$$N_e \cong \sum Z_i N_i \tag{1}$$

where the symbol Σ implies summation from 1 to n. It follows from the Boltzmann equation that

$$\frac{1}{N_i}\frac{dN_i}{ds} = -\frac{A_iMg(s)}{kT} + Z_i\frac{eE(s)}{kT} \tag{2}$$

* Aller and Chapman (*14*) have recently discussed the consequent reduction of the abundance of heavy elements in the solar atmosphere. It has been pointed out to us by Dr. A. J. Dessler that an interesting application of this stratification has been made recently to the separation of O^+ and H^+ ions in the outer terrestrial atmosphere by Mange (*15*).

$$\frac{1}{N_e}\frac{dN_e}{ds} = -\frac{mg(s)}{kT} - \frac{eE(s)}{kT} \tag{3}$$

for hydrostatic equilibrium. The electric field $E(s)$ is the result of a very slight electron-ion separation $(\Delta N/N) = 0[(Mg/e)^2/NkT]$ and is taken to be positive in the upward direction. In the solution of (3) we shall drop the electron mass m. The reader may carry it along if he wishes, but it contributes nothing of interest to the end result.

Solution of Eqs. (1–3) is easily reduced to solution of a polynomial in N_i. We multiply (3) by Z_i and add to (2), thereby eliminating $E(s)$. The equation may be integrated to give $N_i(s)$ in terms of $N_e(s)$,

$$N_i(s) = N_i(0)\left[\frac{N_e(0)}{N_e(s)}\right]^{Z_i} \exp\left[-A_i h(s)\right], \tag{4}$$

where $h(s)$ is the height s measured in units of the basic scale height $kT/Mg(s)$,

$$h(s) = \int_0^s ds\, \frac{Mg(s)}{kT}.$$

On the other hand, instead of integrating the result, we may instead multiply by Z_j and subtract from the same equation with i and j interchanged, which can then be integrated to give

$$\left[\frac{N_i(s)}{N_i(0)}\right]^{Z_j} \exp A_i Z_j h(s) = \left[\frac{N_j(s)}{N_j(0)}\right]^{Z_i} \exp A_j Z_i h(s). \tag{5}$$

Using (1) to express $N_e(s)$ in terms of $N_i(s)$, and (5) to express $N_i(s)$ in terms of the density of one specific ion species $N_\alpha(s)$ we obtain from (4), with $i = \alpha$,

$$\sum Z_j N_j(0) = \left[\frac{N_\alpha(s)}{N_\alpha(0)}\right]^{1/Z_\alpha} \exp \frac{A_\alpha h(s)}{Z_\alpha} \times \sum Z_j N_j(0)\left[\frac{N_\alpha(s)}{N_\alpha(0)}\right]^{Z_j/Z_\alpha} \exp \frac{A_\alpha Z_j - A_j Z_\alpha}{Z_\alpha} h(s), \tag{6}$$

which is a polynomial in $[N_\alpha(s)/N_\alpha(0)]^{1/Z_\alpha}$ of a degree equal to the maximum Z_i plus one, say $Z_n + 1$. The expression may also be viewed as a polynomial in $\exp h(s)$. If hydrogen is a constituent it is probably convenient to choose it as the reference ion, so that $Z_\alpha = A_\alpha = 1$.

To illustrate the separate concentration of different ionic species, consider an atmosphere composed only of hydrogen and helium. Then if N_1 is the hydrogen density and N_2 the helium density, we have $Z_1 = A_1 = 1, Z_2 = 2, A_2 = 4$. Then (6) may be written

$$\xi^3 \exp[-2(h - h_c)] - 2\exp[-(h - h_c)] + \xi^2 = 0 \tag{7}$$

where

$$\frac{N_1(s)}{N_1(0)} = \frac{[1 + 2N_2(0)/N_1(0)]^{2/5}}{2^{3/5}[N_2(0)/N_1(0)]^{1/5}}\,\xi \tag{8}$$

and

$$h_c = \frac{1}{5}\ln\{2[N_2(0)/N_1(0)]^2[1 + 2N_2(0)/N_1(0)]\}. \tag{9}$$

We see that (7) is a cubic in $N_1(s)$ or a quadratic in $\exp[-(h - h_c)]$. It is easiest to solve for $\exp[-(h - h_c)]$,

$$\exp[-(h - h_c)] = \frac{1}{\xi^3}\{1 \pm (1 - \xi^5)^{1/2}\}. \tag{10}$$

We require that $\xi \to 0$ as $h \to +\infty$ so that above (i.e. $h > h_c$) the branch point ($h = h_c$, $\xi = 1$) the $\pm$ must be minus. The maximum value of ξ is one, attained at the branch point. As $h \to -\infty$, the left-hand side of (10) is real and positive, and becomes large without bound. In order that the right-hand side do likewise, it is *necessary* that $\xi \to 0$ and $\pm$ be plus. Altogether, then we must choose $\pm$ as minus when $h > h_c$ and $\pm$ as plus when $h < h_c$. The critical height $h(s) = h_c$ is above $s = 0$ if $N_2(0) > N_1(0)/2$ and below if $N_2(0) < N_1(0)/2$.

The helium density may be written from (5) as

$$\zeta = \xi^2 \exp - 2(h - h_c) \tag{11}$$

where

$$\frac{N_2(s)}{N_2(0)} = \frac{[1 + 2N_2(0)/N_1(0)]^{2/5}}{2^{8/5}[N_2(0)/N_1(0)]^{6/5}}\,\zeta. \tag{12}$$

A consideration of the asymptotic relations is instructive. High in the atmosphere ($h - h_c \gg 1$) (10) and (11) yield

$$\xi \sim 2^{1/2}\exp\left[-\frac{1}{2}(h - h_c)\right], \quad \zeta \sim 2\exp[-3(h - h_c)].$$

The atmosphere is essentially pure hydrogen with the conventional scale height $2kT/Mg$. The helium is dropping off much more rapidly than the normal $3kT/4Mg$ scale height. Deep in the atmosphere ($h - h_c \ll -1$)

$$\xi \sim 2^{1/3}\exp\left[\frac{1}{3}(h - h_c)\right], \quad \zeta \sim 2^{2/3}\exp\left[-\frac{4}{3}(h - h_c)\right].$$

The atmosphere is essentially pure helium with the normal scale height $3kT/4Mg$. The slight hydrogen contamination is increasing upward at a very slow rate corresponding to a scale height $-3kT/Mg$.

The hydrogen and helium densities, ξ and ζ, are plotted in Fig. 1 along with the asymptotic curves. We see that the ionized hydrogen essentially floats on top of the ionized helium atmosphere. The hydrogen density decreases rapidly (exponentially) in the downward direction below the critical level $h = h_c$, and the helium cuts off in the upward direction above $h = h_c$ with a scale height much shorter than in the normal ionized helium atmosphere below $h = h_c$. This is in sharp contrast to an atmosphere of neutral hydrogen and helium, where both constituents increase exponentially downward with unvarying scale heights.

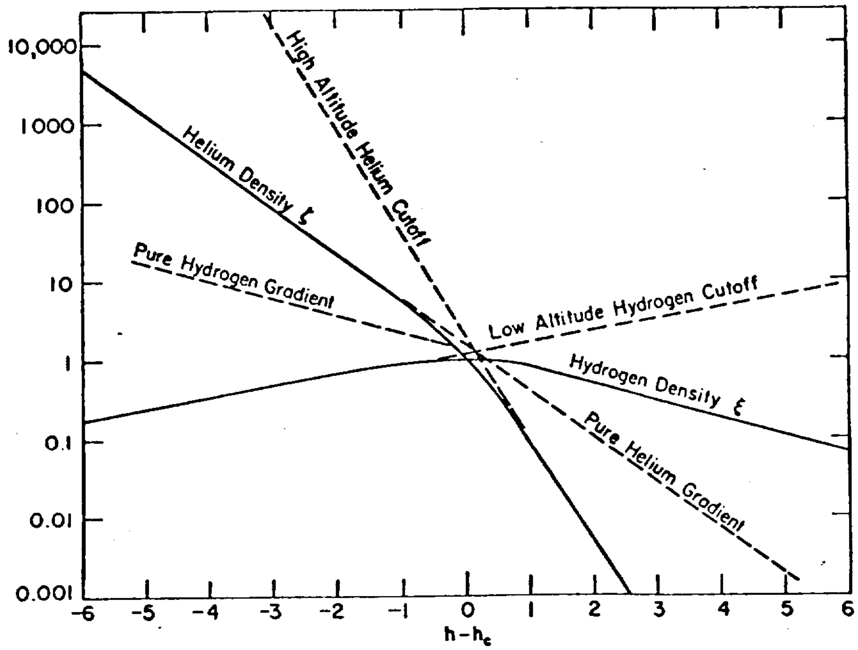

FIG. 1. Plot of the hydrogen and helium densities as a function of scaled height $h(s) - h_c$ in an isothermal ionized hydrogen-helium atmosphere. The asymptotic densities are shown by the broken lines.

For convenience choose the origin of the coordinates so that $s = 0$ at the critical level $h = h_c$. If this is done, then $\xi = N_1(s)/N_1(0)$, $N_2(0) = \frac{1}{2}N_1(0)$, and $h_c = 0$. The density $N_1(0)$ at the critical level is easily related to the total mass of hydrogen in a column of unit cross-section extending from $s = -\infty$ to $s = +\infty$. For instance, if $g(s)$ may

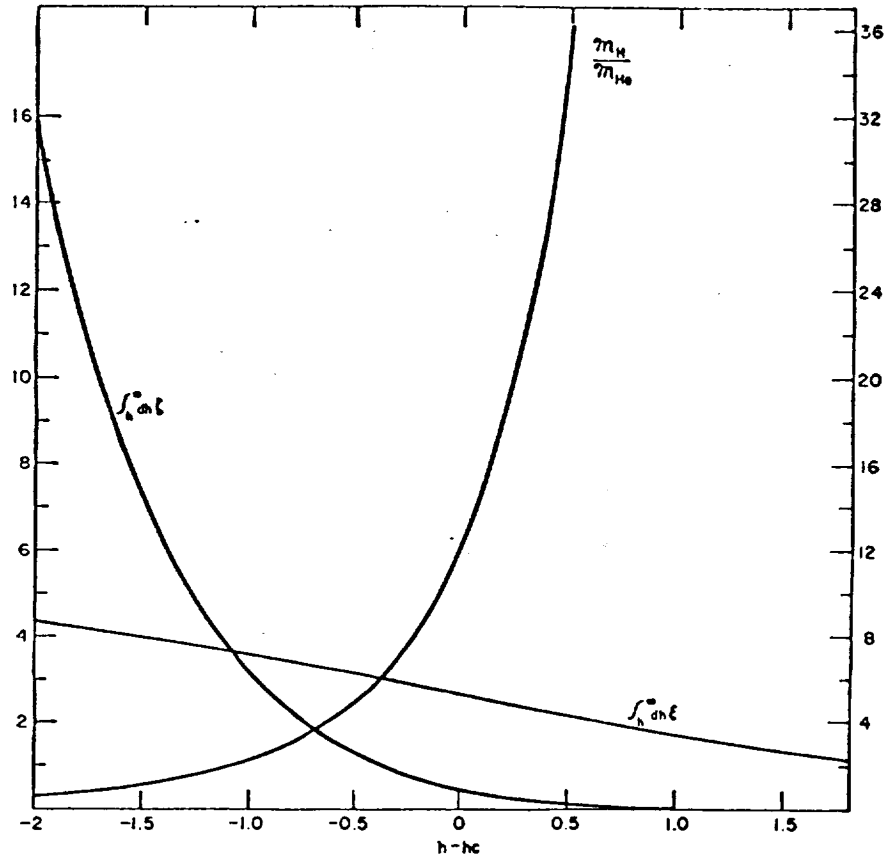

FIG. 2. Plot of $\int_h^\infty dh\,\xi$, $\int_h^\infty dh\,\zeta$, and the number ratio m_H/m_{He} as a function of scaled height for an ionized hydrogen-helium atmosphere. The two integral quantities refer to the vertical scale on the left-hand side, whereas the number ratio refers to the scale on the right.

be taken as uniform with height over the region containing most of the hydrogen, then we define

$$M_H(s) \equiv \int_s^\infty ds\, N_1(s) = \frac{kTN_1(0)}{Mg} \int_h^\infty dh\, \xi,$$

$$M_{He}(s) \equiv \int_s^\infty ds\, N_2(s) = \frac{kTN_1(0)}{2Mg} \int_h^\infty dh\, \zeta.$$

It is readily shown by numerical integration that the total hydrogen number per unit area is

$$M_H(-\infty) = 6.3kTN_1(0)/Mg.$$

In Fig. 2 we plot $\int_h^\infty dh\, \xi$ and $\int_h^\infty dh\, \zeta$ for intermediate values of h. To demonstrate the extent to which the hydrogen floats above the helium we have plotted $M_H(s)/M_{He}(s)$. We find that 0.42 of the total hydrogen is above $h = h_c = 0$, and 0.65 is above the level $h = -1.55$ at which $M_H = M_{He}$.

The most obvious consequence of the tendency of the different ionic species in a stellar atmosphere to stratify is the complication which such stratification would introduce into interpretation of the emission and absorption spectrum of the atmosphere.

Suppose, on the other hand, that a tenuous stellar atmosphere, such as the solar corona, is *not* sufficiently quiet as to allow the separation of ionic species. Then the continual settling of helium through the background hydrogen would result in a very large amount of coronal heating. To demonstrate the importance of this heating in the solar corona if the corona is sufficiently well stirred—and its ionic stratification if it is not—suppose that we suddenly invert a large block of coronal material so that there are more heavy ions at high altitude than would result from equilibrium. Following inversion, the elevated heavy ions drift downward through the background of hydrogen with a mean velocity $\langle v \rangle \cong \frac{1}{2}(\frac{3}{4}g)t_D$ where g is the gravitational acceleration* and t_D is the mean free time between deflecting collisions. At each collision, gravitational energy of the falling heavy ion is transferred to the background hydrogen, producing a heating of $NAMg\,\langle v \rangle$ ergs/cm^3. Presumably helium is the dominant heavy ion, so that $A = 4$ and $N = N_2$. To judge the relative importance of this heating we compare it with the principal radiative loss, free-free emission from hydrogen (assuming that helium is no more abundant than about 0.3 N_1) which is $1.4 \times 10^{-27} N_1^2 T^{1/2}$ ergs/cm^{3}sec (Spitzer, *16*). We have (*16*) $t_D \cong 7 \times 10^{-13}\, w^3/N_1$ for the deflection of fully ionized helium with a thermal velocity w in a hydrogen background of density N_1 (at coronal temperatures so that $\ln \Lambda \cong 20$). At the solar photosphere $g \cong 2.7 \times 10^4$ cm/sec^2; at an altitude of 3×10^4 km above the photosphere, $g \cong 1.3 \times 10^4$ cm/sec^2: We employ the nominal value $g = 2 \times 10^4$

* The effective gravitational acceleration of He^{++} in a hydrogen atmosphere is $3g/4$, as discussed in the next section.

hydrogen atmosphere falls downward with an acceleration $3g/4$. Note however that if $Z/A > (1 + Z')/A'$, then η is positive and the ion will "fall" upward with acceleration $\eta g(s)$. Such an ion is ejected from the stellar atmosphere into interstellar space with a velocity at infinity which is $\eta^{1/2}$ times the gravitational escape velocity $v_e = [2g(a)a]^{1/2}$ from the level at which the ion is released. Consider a hypothetical star whose outer atmosphere is composed predominantly of helium (*17*). Then a hydrogen atom diffusing out to a radial distance a beyond which it makes no more collisions, is projected into space with a velocity increasing outward to $0.58v_e$ at infinity. The energy of such ejected hydrogen ions is of the order of kilovolts.

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